Interactions of detonation waves

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The interactions of discontinuities, in which detonation waves participate, are analysed. For this purpose detonation polars are introduced and their properties are examined. A full list of possible interactions is then given, and the use of the polars for the quantitative solution of intersection problems is illustrated.

1. Introduction

In one-dimensional flow, interaction problems involving detonation waves have been extensively treated. The literature on oblique detonation waves, however, is still very poor. The behaviour of steady-state detonation has been discussed by Rutkowski & Nicholls (1956).† The possibility of stabilizing a detonation wave on an edge has been analysed by the author (1958*a*). It has been suggested by Dunlap, Brehm & Nicholls (1958), and by the author (1958*b*), that a steady-state detonation wave be used for the combustion in a ramjet-type engine.

In the present paper, we examine the various configurations of intersections in which detonation waves participate, together with shock waves, expansion waves and shear discontinuities. The steady-state problem in which the waves 'intersect' one another is essentially equivalent to the non-steady one in which non-parallel waves 'collide'.

The data on detonation, necessary for the analysis, are first presented, and detonation polars are introduced. A complete list of possible intersections, based on the distinction introduced by Landau between incident and emerging waves, is then given; and finally a number of characteristic cases are discussed in detail.

2. Some properties of detonation waves

We start by recalling the features of detonation in gaseous combustible mixtures. An ordinary detonation wave is composed of two distinct zones; its front is a thin shock wave in which the flow is compressed and heated, and combustion then occurs in a much wider zone. This picture is altered when the thickness of the shock wave is for some reason increased, as happens for very weak or very strong shocks. The two zones may then become partially intermixed. (It is of importance that the total width of the wave is small so that its treatment as a surface of discontinuity is appropriate for all practical purposes.)

In a combustible mixture in which burning releases an amount of heat q per unit mass, detonation waves can propagate with all velocities greater than a

† The author is acquainted with this paper only from a reference in Dunlap *et al.* (1958).

characteristic minimum velocity v_J. The wave has this minimum velocity when the detonation process occurs at the Jouguet point on the detonation adiabatic. Jouguet detonation is of particular importance, since spontaneously appearing waves, or those in a flow on which sufficiently weak constraints are imposed, are always of this sort.

In a frame of reference in which a Jouguet wave in an ideal gas is at rest, the following relations hold for the normal detonation wave:

$$\left.\begin{aligned}
&(a)\ v_1 = v_J = a_1 M_J = \sqrt{\left(\frac{\kappa_2-1}{2}\left[(\kappa_2+1)\,q + \frac{(\kappa_1+\kappa_2)\,p_1}{(\kappa_1-1)\,\rho_1}\right]\right)} \\
&\qquad\qquad + \sqrt{\left(\frac{\kappa_2+1}{2}\left[(\kappa_2-1)\,q + \frac{(\kappa_2-\kappa_1)\,p_1}{(\kappa_1-1)\,\rho_1}\right]\right)}, \\
&(b)\ \frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = V = \frac{1+1/(\kappa_1 M_J^2)}{1+1/\kappa_2}, \quad (c)\ \frac{p_2}{p_1} = \frac{\kappa_1 M_J^2+1}{\kappa_2+1}
\end{aligned}\right\} \tag{1}$$

More generally, the velocity v_D of the detonation wave is greater than v_J, owing to the process not taking place at the Jouguet point. The formulae are now more complicated and have the form:

$$\begin{aligned}
&(a)\ \frac{v_2}{v_D} = V = \frac{\kappa_2}{\kappa_2+1}\left(1+\frac{1}{\kappa_1 M_D^2}\right) - \frac{\kappa_2}{\kappa_2+1} \\
&\qquad \times \sqrt{\left(\left(1+\frac{1}{\kappa_1 M_D^2}\right)^2 - \frac{\kappa_2^2-1}{\kappa_2^2}\left\{1+\frac{M_J^2}{M_D^2}\left[\frac{\kappa_2^2}{\kappa_2^2-1}\left(1+\frac{1}{\kappa_1 M_J^2}\right)^2 - 1\right]\right\}\right)}, \\
&(b)\ M_2^2 = \frac{V}{\kappa_2(1-V+1/(\kappa_1 M_D^2))}, \quad (c)\ \frac{p_2}{p_1} = \frac{\kappa_1 M_D^2+1}{\kappa_2 M_2^2+1}.
\end{aligned} \tag{2}$$

In the above relations, v is the velocity, M the Mach number, p the pressure, ρ the density, and κ the adiabatic coefficient. Indices 1 and 2 have been used to denote the gas before and after detonation.

In a flow of a combustible gas, the velocity v_J is a function of position. We now introduce the locally defined number

$$J = \frac{v}{v_J}, \tag{3}$$

which plays a role in the geometry of detonation waves analogous to that of the Mach number for shocks in inert gases. In regions of flow where $J > 1$, stationary (oblique) detonation waves, propagating with a velocity $v \geqslant v_D \geqslant v_J$ can be produced. They are inclined to the streamlines at an angle δ given by

$$\sin\delta = \frac{v_D}{v} = \frac{1}{J}\cdot\frac{v_D}{v_J} \tag{4}$$

To obtain relations for oblique detonation waves, a tangential velocity component is superimposed on the normal wave. In the oblique detonation wave, the flow is rotated through an angle β (figure 1) given by

$$\tan\beta = \frac{(1-V)\tan\delta}{1-V\tan^2\delta}. \tag{5}$$

For the solution of problems involving oblique detonation waves it is opportune to introduce the detonation polars. In the v_{2y}–v_{2x} plane, we have

$$\left.\begin{aligned} \tan\delta &= \frac{v_1 - v_{2x}}{v_{2y}}, \\ V &= \frac{v_{2x}\sin\delta - v_{2y}\cos\delta}{v_1\sin\delta}. \end{aligned}\right\} \tag{6}$$

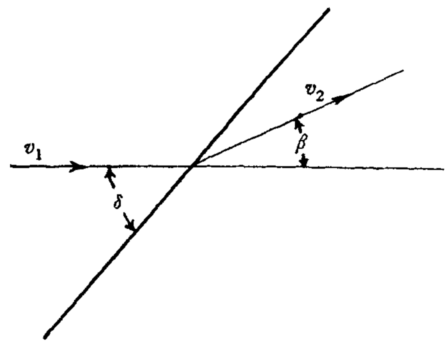


FIGURE 1. The oblique detonation wave.

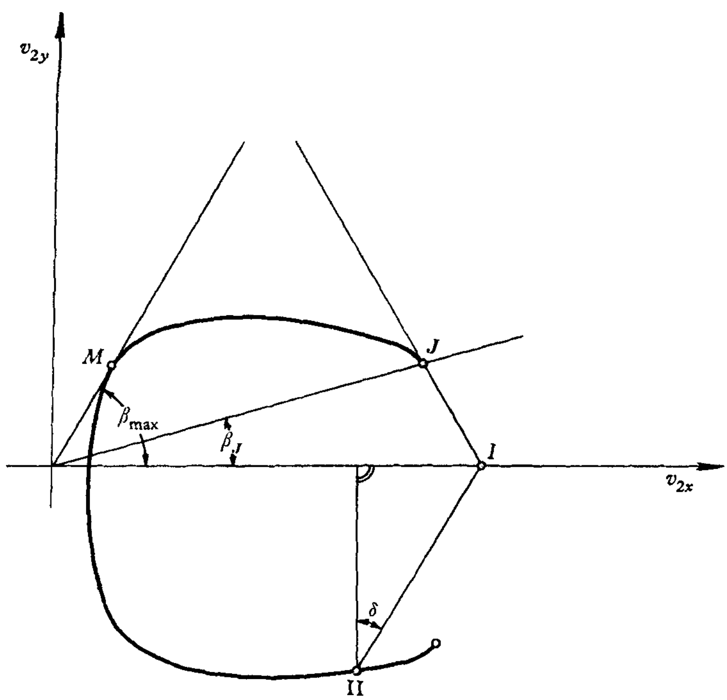


FIGURE 2. The v_{2y}–v_{2x} polar.

These equations define the polar parametrically, the parameter being the slope δ of the wave. (V depends on δ only.) The angle δ has the usual geometric representation (see figure 2). The expression $f(v_{2x}, v_{2y}) = 0$, which is an algebraic curve of the sixth degree, is long and we do not give it here. On this curve, the Jouguet detonation is represented by the point J. Only the part on the left-hand side from (J) of the curve (6) represents detonation waves, so that the detonation polar is an open curve.

The radical in formula (6) which depends only on δ vanishes for $M_D = M_J$; hence the angle δ is a minimum at the point J. It is easily seen from this that the tangent at J to the polar passes through the point I, which represents the unburned gas. We also remark that $(d\delta/d\beta)_J = 0$. This is typical of detonation; and when $M_J - 1 \to 0$ so that the point J approaches I, $(d\delta/d\beta)_J \neq 0$. The detonation polar is always completely enclosed by the shock polar. For values of $J \gg 1$, the detonation polar approaches the shock polar for a gas moving at a Mach number M_1/J_1.

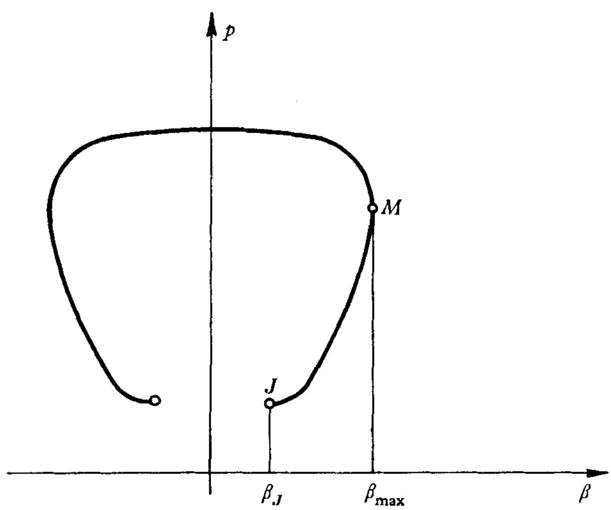


FIGURE 3. The p–β polar.

The point M, at which maximum deviation of the flow ($\beta = \beta_{max}$) is obtained, divides the polar into a region of 'weak' and one of 'strong' waves. Deviations of the flow through angles between β_J and β_{max} can be achieved by both weak and strong waves. A deviation through an angle less than β_J is not possible by a weak wave, as there is no detonation wave with Mach number less than M_J. We expect the flow pattern with detonation waves to tend, for vanishing heat release, to the usual one with shock waves. In order to achieve this correspondence, the weak wave should be chosen when possible. More details and numerical results for the flow with oblique detonation waves can be found in Larisch (1958*a*).

We shall also need the polar in the p–β plane. It is given parametrically by equations (2*c*) and (5), and its appearance is as in figure 3.

We have seen that at the Jouguet points, $d\delta/d\beta = 0$. We must have there also $dp/d\beta = 0$, so that the pressure is minimum.

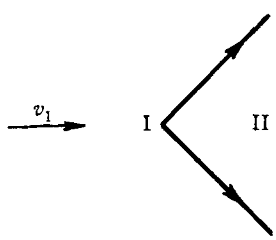
Shock waves can, of course, propagate in a combustible mixture; but their intensity must be moderate, so that two conditions have to be satisfied. First, the mixture has not to be at ignition conditions behind the shock. Secondly, as can be seen from (3), the number J falls in a shock (as v decreases, v_J increases) and in a sufficiently strong shock becomes less than unity. A flow with $J < 1$ should, however, be considered as unstable with respect to the spontaneous appearance of detonation and in the stable flow configuration the shock is replaced by a detonation wave. Shock waves which satisfy these two restrictions will be called 'permitted'

3. Classification of intersections

We now examine the cases in which detonation waves, shock waves and shear discontinuities interact.

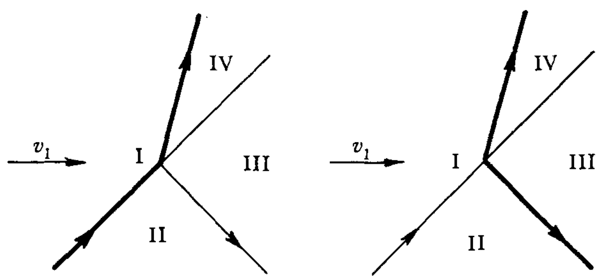
As usual, only a small zone surrounding a point on the line of intersection will be considered; this line can then be assumed to be straight. The motion, in a conveniently chosen frame of reference, is steady and lies in a plane normal to the intersection line. The flow pattern is obviously composed of sectors of parallel flow limited by centred expansion waves and various discontinuity surfaces.

In the treatment proposed by Landau for shock intersections, which we follow in our exposition, the distinction between 'incident' and 'emerging' waves is of chief importance. Waves in which the tangential component of the velocity is directed toward the intersection line are called incident; in the opposite case



Case A1

FIGURE 4



Case B1 Case B2

FIGURE 5

FIGURE 4. Case A, no incident wave. Spontaneous detonation.

FIGURE 5. Case B, one incident wave. Case B1, splitting of a detonation wave. Case B2, splitting of a shock wave.

they are called emerging. Since the incident waves are produced by factors which do not depend on the intersection itself, the meeting of more than two such waves should happen only exceptionally. The same must be said of more than one incident wave intersecting a particular surface at a given point.

The known rule, asserting that a streamline cannot cross more than one emergent wave, has, in our case, an exception. Jouguet waves travel through the burned gas with sound velocity (see (1)), so that if there is such a wave among the emerging ones, it is generally continued by an expansion wave.

In zones where a combustible gas flows, we always assume that $J > 1$, and also that the burning, which occurs after the interaction, takes place in a detonation wave. Shock waves in the unburned zone are supposed to have what we have called a 'permitted intensity'.

We can now examine all possible cases of interactions. On the diagrams, fat arrows are used to denote detonation waves, thin arrows for shock waves, thin lines for shear discontinuities and dotted lines for the boundaries of expansion

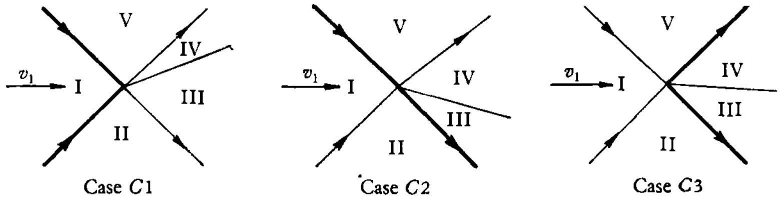


FIGURE 6. Case C, two incident waves. Intersections of waves from different families, cases $C\,1$, $C\,2$, $C\,3$.

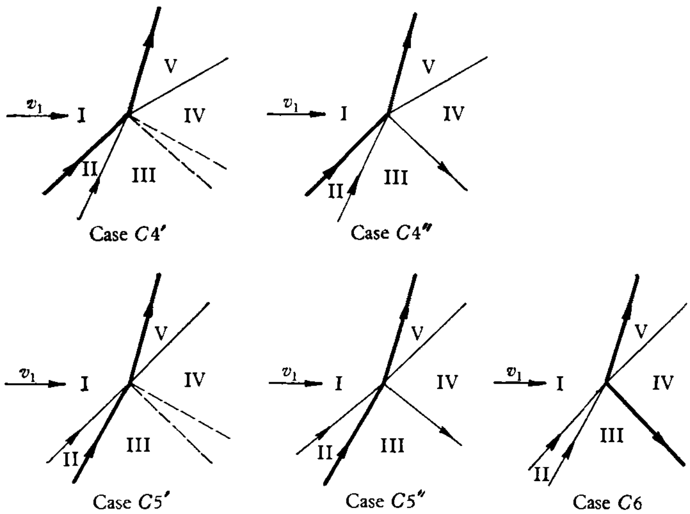


FIGURE 7. Case C, two incident waves. Intersections of waves from the same family, cases $C\,4'$, $C\,4''$, $C\,5'$, $C\,5''$, $C\,6$.

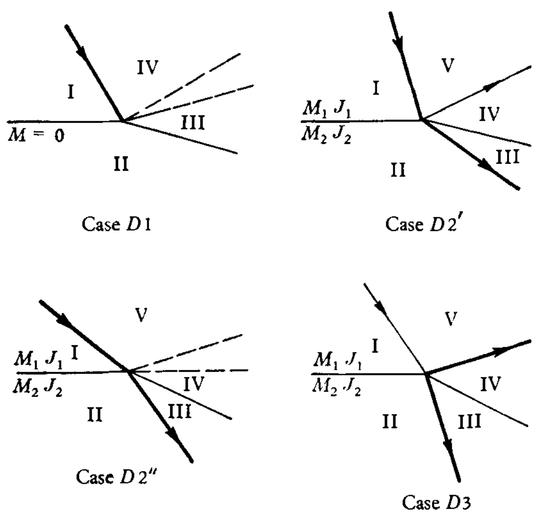


FIGURE 8. Case D, intersections with a discontinuity surface. Case $D\,1$, reflexion from a zone of subsonic flow. Cases $D\,2'$ and $D\,2''$, interaction of a detonation wave with a shear discontinuity. Case $D\,3$, interaction of a shock wave with a shear discontinuity.

fans. Expansion waves, which occur behind emergent Jouguet waves, are not specified. The intersections are classified by the number of incident waves which interact. At the end, the possible configurations for a wave intersecting a shear discontinuity surface are given.

We now briefly discuss each interaction separately. Detonation can occur in a flow spontaneously; accordingly, there is a case of intersection with no incident wave and two emerging Jouguet waves (case $A\,1$), while there are two intersections possible with one incident wave. The diagram of case $B\,1$ represents a detonation wave splitting into a more intense one and a shock wave. The inverse case, with the emerging wave weaker than the incident one and with a reflected expansion wave instead of the shock, could be observed only if the incident wave were from the strong family and is therefore not of interest. On the diagram of case $B\,2$ a shock wave is splitting and there are two emerging detonation waves.

Two types of intersections with two incident waves are possible, according as the waves deflect the flow in opposite directions or in the same one. All configurations may be met, the pair of incident waves being composed of two detonation waves, a detonation wave and a shock wave, or two shock waves. As to the intensity of the incident waves, the cases $C\,4$ and $C\,5$ might occur in two variants, with an expansion wave or a shock wave being reflected.

Three schemes might be expected to occur in the interaction with a discontinuity surface. In the case $D\,1$ the flow is subsonic on one side of the surface. The configuration of case $D\,2$ is possible in the usual two variants. In the case $D\,3$, the detonation is initiated by a shock wave. In degenerate cases the flow is inert on one side of the surface; the corresponding detonation wave is then replaced by a shock wave.

When the two incident waves are of the same intensity, the case $C\,1$ represents also the reflexion of a steady-state detonation wave from a rigid surface. It is known, however, from the case of shock waves, that this formal solution for all the cases D is to be used with care, in a steady-state motion, owing to the interaction with the boundary layer which can change the situation radically.

4. On the quantitative analysis of interactions

In order to determine the parameters of the emerging waves and those of the flow in the various zones, we make use of the fact that the pressure and the direction of the flow are the same on both sides of a shear discontinuity. The problem of finding out the intersections of the corresponding p, β polars involves here a much more intricate algebra than in the cases when shock waves only interact. There are three given parameters for the cases B, four for the cases C and five for the cases D. Obviously, solutions will exist only for a part of the domain in which these parameters are defined.

We now give indications of the way in which the most typical situations can be analysed.

It was mentioned that emerging Jouguet waves are continued by expansions. When emerging waves are looked for, the detonation polars are conveniently drawn with the 'expansion polars' which are their continuations from the Jouguet point.

The polars for case $B2$ are drawn on figure 9. The configuration starts to be possible formally, when the p, β polar of zone II intersects the polar of zone I on its top. In this situation the intensity of the splitting shock wave is a minimum. The splitting would be actually observable only if this intensity be 'permitted'.

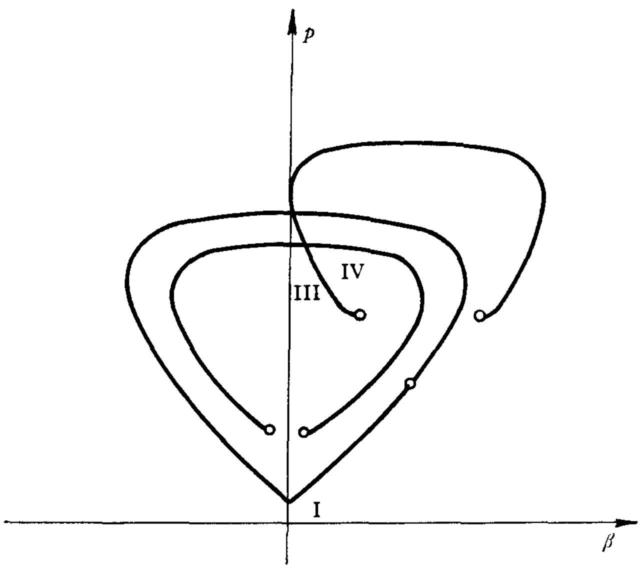


FIGURE 9. Polar diagram for case $B2$.

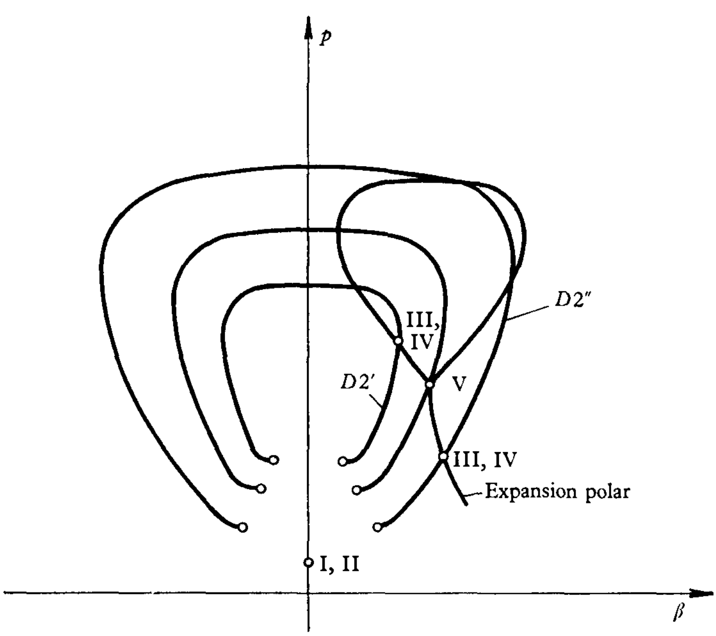


FIGURE 10. Polar diagram for cases $C4$

It is of importance to determine when the regular reflexion of detonation waves (a particular case of $C1$) is possible. This situation has been investigated by Staniukovich (1955). When there is an incident Jouguet wave, and $\kappa = 3$, the calculations show that regular reflexion is possible as long as $\delta \gtrsim 70°$.

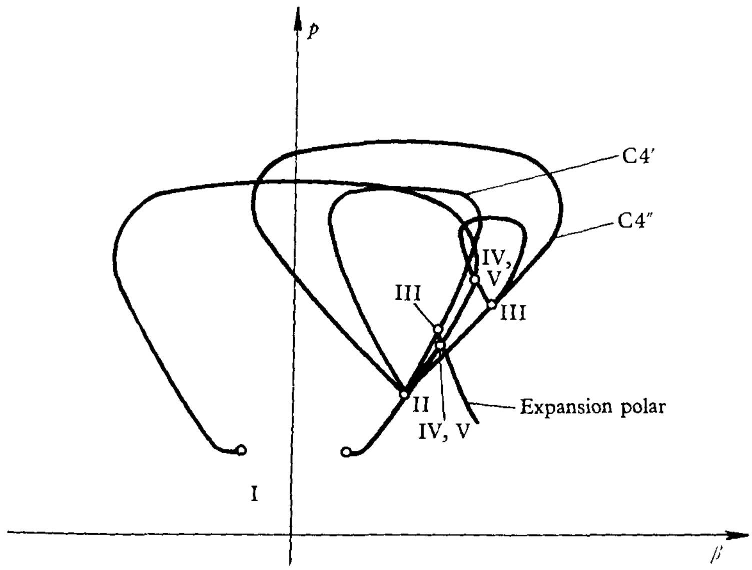


FIGURE 11. Polar diagram for cases $D2$.

In figure 10, the polar diagram for the cases $C4$ is given. The reflected wave is an expansion or a shock according as the point III lies inside or outside the detonation polar of zone I. The results are similar for the interaction case $C5$.

Finally, we show in figure 11 the polar diagram for the cases $D2$. Similar criteria as before apply as to the nature of the reflected wave. It is a shock when the detonation polar of zone II lies inside that of zone I, and an expansion in the other case.

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